steel, fig. 4. They described the precipitates as glassy silica inclusions and from their discussion it is evident that they regard them as vitreous silica. This opinion seems to be shared by a number of authors who have later reported on this type of inclusion. It is sometimes referred to as raspberry-like [8].

Torssell [9] accepted the view that the raspberry-like precipitate in steel consists of vitreous silica and pointed out the importance of its high viscosity, as outlined above. However, he did not draw the conclusion that the same principles should hold for the case of precipitation of silica rosettes from the non-metallic matrix of slag inclusions in the steel. He still accepted them as cristobalite.

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A Fracture of Rubber in a State of Finite Torsional Shear

An interesting biaxial mode of failure has been observed in a solid rubber cylinder which had metal end plates bonded on to the flat ends.

Failure in torsion by a crack propagating perpendicular to the maximum tensile stress is well known for many materials, but biaxial failure in torsion has not, within the knowledge of the authors, been previously reported for a rubber.

The failure occurred during studies of the stress distribution necessary to maintain the rubber in a state of finite torsional shear. The rubber was an unfilled natural rubber, vulcanised for 30 min at 150° C using 5% by weight of dicumyl peroxide. The rubber cylinder was 4 in.* in diameter and 0.57 in. long.

It has been shown that the surface tractions necessary to maintain a right circular cylinder in a state of finite torsional shear correspond to two sets of stress components each acting only over the plane ends of the cylinder [1]. In addition to the distribution of tangential surface tractions providing the twisting couple, a distribution of normal surface tractions is required to maintain the cylinder at constant length. In the work described in this note the torsional shear was maintained by mounting the rubber unit vertically in such a manner as to allow the bottom plate to be twisted with respect to the top plate, while maintaining the length constant to an accuracy of better than 0.05% [2].

Fracture was observed in a particular rubber specimen when one end of the unit was twisted through 25° with respect to the other end. The failure was unintentional. Two distinct types of fracture surfaces were created, a primary surface and a secondary surface, and the nature of these surfaces has been studied. The directions of the fracture path have been examined in terms of the stress distribution through the rubber.

A limited number of attempts were made to reproduce the complete mode of failure by deliberately initiating a fracture in other specimens of rubber in a similar state of shear. Only one type of fracture surface was created, the nature and direction corresponding to the relatively smooth primary fracture surfaces discussed below.

When failure occurred the shear strain at the curved surface of the rubber cylinder was about 150% (i.e. a shear angle of about 56°). The deformation was then removed. The fracture path on the curved surface of the undeformed rubber

*1 in. = 2·54 cm.

can be seen in fig. 1. The deformation had been imposed by twisting the bottom plate DD^1 from left to right with respect to the top plate A^1A.

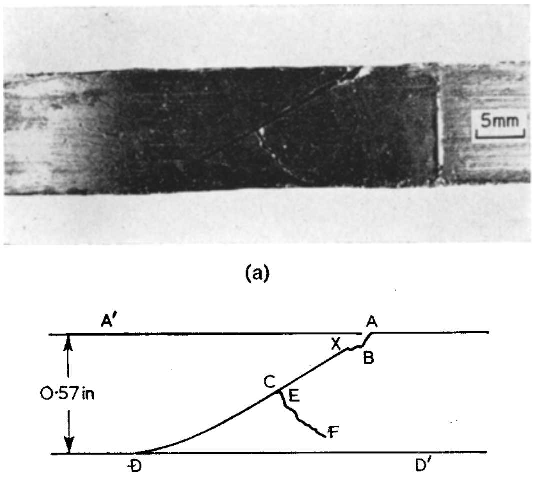

Figure 1 Fracture path on the surface of the undeformed rubber cylinder.

The fracture was clearly initiated at A. The short line AB is at 45° to the direction of the twist and therefore in the deformed state AB was perpendicular to the principal tensile stress. This is the direction the initial part of a fracture path would be expected to follow. The initial part of the fracture CF is perpendicular to AD at C. In attempting to understand the nature of the fracture it is necessary to consider the directions of the fracture path in the deformed rubber. These directions are shown in fig. 2, together with the principal stress directions at the instant before failure.

A possible order in which the complete fracture was generated is as follows. Consider fig. 2b. The crack was initiated at A, in a direction AB′ perpendicular to the maximum tensile stress and the direction then changed to B′X′. The abrupt change in direction presumably occurred at a flaw. The newly created surfaces along B′X′ are rough which suggests that this part of the fracture was at a rate of propagation slow enough to allow rubber to behave as an elastomer rather than as a glass-like brittle material. The crack then propagated rapidly along X′D′, creating comparatively smooth glass-like primary surfaces.

It is probable, from considerations of the Griffith fracture criterion in three dimensions, that X′D′, and hence the complete primary surface, is perpendicular to the new principal tensile stress direction after local stress redistribution at the crack tip. The Griffith criterion for a critical fracture stress in a uniaxial stress system has been extended to two- and three-dimensional systems by a number of workers and discussed by Andrews [3]. He reports that if the principal stresses σ_1 and σ_2 are such that $3\sigma_1 + \sigma_2 > 0$, then the most favoured direction for crack-propagation is perpendicular to the principal tensile stress σ_1. We will now show that the

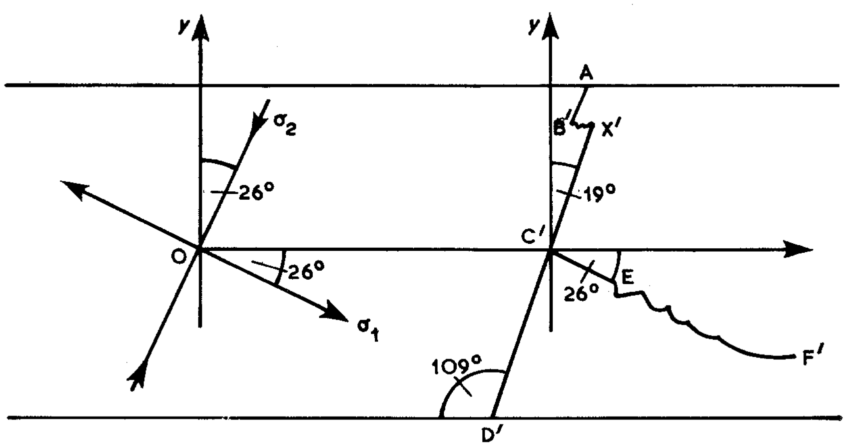

Figure 2 (a) The principal stress directions on the rubber surface at the moment before fracture. (b) The position of the cracks on the curved surface when the rubber cylinder is in the deformed state.

criterion $3\sigma_1 + \sigma_2 > 0$ is obeyed for a solid rubber cylinder in a state of finite torsional shear.

If we assume that W, the elastic free energy which is stored in the rubber during deformation, can be described by the kinetic theory of rubber-like elasticity, then $W = C_1 (I_1 - 3)$, where C_1 is a material parameter related to the shear modulus of the rubber, and I_1 is a strain invariant [4].The stress matrix (σ_{ij}) for finite simple shear in an incompressible material has been used derived by Rivlin [1] for a completely general form of W. If W is given by $W = C_1 (I_1 = 3)$ then it can be shown that (σ_{ij}) is given by

$$(\sigma_{ij}) = \begin{pmatrix} 2\psi^2 r C_1 & 2\psi r C_1 & 0 \\ 2\psi r C_1 & 0 & 0 \\ 0 & 0 & 0 \end{pmatrix} \quad (i, j, = 1, 2 \text{ or } 3)$$

where r is the radial displacement from the cylinder axis and ψ is the angular twist per unit length. The local co-ordinate axes $0x_1$ and $0x_2$ are in the directions tangential to the curved cylinder surface, and in the direction of the cylinder axis respectively. If the axes are rotated by an angle α about the $0x_3$ axis (the radial direction), such that the stress matrix (σ_{ij}) is transformed to (σ'_{ij}), then it can be shown that:

$$3\sigma'_{11} + \sigma'_{22} = 4\psi r C_1 \sin 2\alpha + 2\psi^2 r^2 C_1 .$$

When α is given by $(\psi a) \tan 2\alpha = 2$, where a is the cylinder radius, the new axes are coincident with the principal axes and σ'_{11} and σ'_{22} are the principal stresses σ_1 and σ_2 respectively. Then at the surface shear strain of 150% ($\psi a = 1.5$) at which fracture occurred it can be seen that $\sigma_1 + 3\sigma_2 > 0$.

Examination of the fractured rubber cylinder showed that the primary fracture surface bounded by X′D′ only penetrated about a quarter of the way towards the axis of the cylinder. The bulk of the cylinder therefore remained stressed after creation of these surfaces. It is therefore likely that the crack X′D′ was opened out by these residual stresses because the cylinder remained in its twisted state. The stress system in the new surface then included a tensile stress component which could be a maximum at the cylinder surface, presumably at C which is midway between the end plates. A second crack then propagated. CE, the first part of the edge of the secondary crack, is perpendicular to the primary fracture. The roughness of the newly created secondary surfaces along CEF suggested considerable stress-reorientation as the fracture propagated as a consequence of a comparatively low rate of propagation.

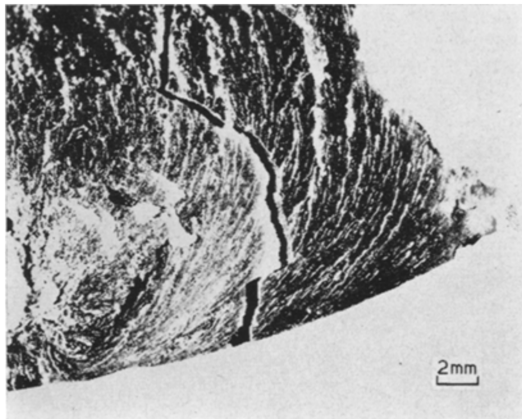

Figure 3 One of the matching fracture surfaces bounded by the edge AD.

The major surface markings on one of the undeformed primary fracture faces bounded by AD (see fig. 1) can be seen in fig. 3. The edge of the secondary fracture surface is clearly identifiable. Examination of this primary surface, and its matching surface, revealed two distinct sets of markings. The major curved markings are clearly visible in fig. 3.

In addition, finer markings could be discerned which appeared to be at something like 30° to these major markings. Due to the low reflectivity of the fracture surfaces however, photography of these fine markings was extremely difficult and therefore a gelatine replica of the surface was prepared, using a 25% solution of gelatine in water. A replica approximately $\frac{1}{16}$ in. thick was produced, mechanically stripped from the matching surface to that shown in fig. 3, lightly shadowed with platinum and then examined in transmitted light in a Vickers projection microscope. These fine markings are shown in fig. 4 as fine cross-markings. From an examination of a series of similar photographs taken from different parts of the primary fracture surface and at higher magnification we believe that the fracture surface consists of low-profile elongated facets and fine steps. The general featureless appearance of the surface between steps indicates that the fracture has progressed in a manner reminiscent of a cleavage type failure.

Burghard and Stoloff [5] in their paper on the fractography of cleavage failures have published a fractograph of a cleavage failure in steel which is very similar to fig. 4. They also state that facets

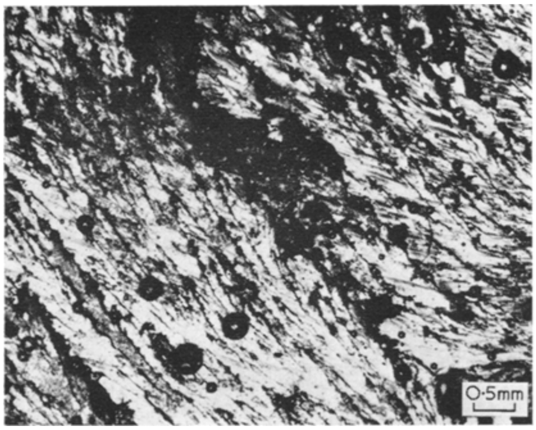

Figure 4 Secondary markings on the main fracture surface.

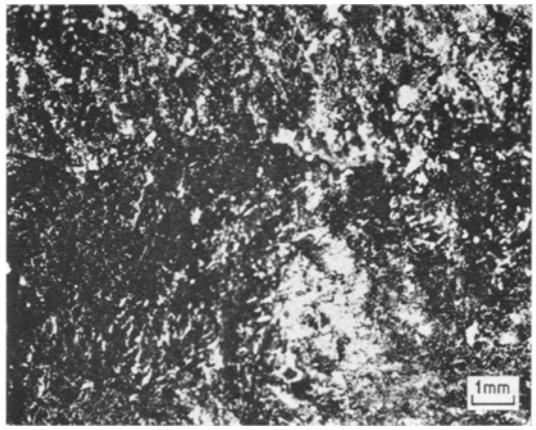

Figure 5 Surface of the secondary fracture.

of the type shown are generally associated with cleavage involving extensive local deformation.

It is well known that at high deformations natural rubber becomes partially crystalline [4]. Therefore a possible explanation of the observed fracture characteristics is that the localised high strain ahead of the progressing crack causes crystallisation and that failure is then through material which can undergo cleavage. It should be noted, however, that this places a restriction upon the crack-propagation rate because of the time necessary to allow for the development of the local crystalline regions. The size of the facets on the rubber fracture are larger, by a factor of several hundreds, than those in the steel fractograph.

The secondary fracture face (CEF in fig. 1) shows quite different characteristics (fig. 5). The surface is generally rough and no features of the type discussed above are to be seen and it appears to be a typical tearing fracture.

We may summarise by noting that a comparatively smooth fracture surface was created in a direction perpendicular to the principal tensile stress and a rough surface was created perpendicular to the principal comprehensive stress. The markings on the smooth surface have some features similar to a cleavage-type failure in a crystalline material. It is possible however that the presence of the secondary tear type of crack is fortuitous, having started as a flaw at F′ (fig. 2) and propagating towards C′, within the time scale of propagation of X′D′.

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Variations in the Lattice Constant of Calcium Oxide

The paucity of reliable data for lattice constant changes in ionic microcrystals is due to the experimental difficulties involved in their measurement, e.g. the preparation of microcrystals sufficiently small to exhibit lattice changes, coupled with the instantaneous adsorption of gases by vacuum calcined powders when exposed to air. In the absence of adsorbed gases, the lattice constant of MgO has been shown to contract with decreasing crystallite size [1]. The magnitude of this contraction is in agreement